Exercises

Optimization with side constraints

Exercise 1. Find all possible extrema (i.e. all points that satisfy the necessary Lagrange condition for extrema) of the following optimization problems:

(a)
$$\max f(x_1, x_2) = \frac{1}{2}x_1^2 + x_2^2 + 2x_2 + 1000$$
$$\text{s.t. } x_1 + x_2 = 80$$

(b)
$$\max f(x, y, z) = x + y + z$$
$$\text{s.t. } xyz - 8 = 0$$

(c)
$$\max f(x, y) = 5x + 8y$$
$$\text{s.t. } xy = 1000$$

Exercise 2. A company has two different, independent production lines. The profit of production line i is a function of the budget spent for the production line x_i:
$$P_1(x_1) = 120\sqrt{x_1} \qquad P_2(x_2) = 160\sqrt{x_2}.$$
The total profit is $P(x_1, x_2) = P_1(x_1) + P_2(x_2)$. Assume the company has total budget of 4 million euro.

- Compute the optimal distribution of the budget between the production lines such that the profit is maximal. Use the Lagrange function (you may assume that every possible extremum is a maximum).
- The Lagrange multiplier λ can be interpreted as the (approximative) change of the objective function if the right hand side of the corresponding side constraint is altered by 1, i.e. if the right hand side changes by Δ then the objective function changes by $-\lambda \cdot \Delta$.
  Assume the budget is reduced to 3.9 million: How does the profit change approximatively?